Chapter 28 Research on Spare Satellites Strategy of Navigation Constellation Based on System Availability

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Abstract The satellite reliability and the service performance of the navigation system will both decrease gradually as the satellite on-orbit operation time increases. In order to guarantee the system always in a stable and reliable operation status, the satellites strategy is necessary. Learning from GPS experience, an improved satellite reliability model is proposed, and then the system availability model is constructed by using the Bayesian network. Finally, according to the requirement of the system availability, the rational spare satellites strategy which includes the spare mode, the spare satellite number, and the spare satellite launch plan is developed. Moreover, the simulation results can provide an important technical support for engineering decisions.

Keywords Navigation constellation · System availability · Spare satellites strategy - Satellite reliability - Bayesian network

28.1 Introduction

The satellite navigation system plays an increasingly significant role in national defence construction and economic development. Thus it is crucial to ensure the system working in stable operation [[1\]](#page-11-0). The spare satellites strategy directly affects

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the realization of system availability, continuity and integrity. It is also the key factor to ensure the stability of the system when some satellite fails.

According to the experience of the construction and operation of GPS and GLONASS, spare satellites are necessary to ensure the system working in continuous and stable operation. The designed GPS constellation consists of 24 satellites, but since the system runs from 1993, the actual number of satellites is always more than 27, which effectively guarantees the system performance. In comparison, because of financial problems, the full constellation of GLONASS is not timely completed, resulting in system performance declining and loosing the broad application market. According to the constellation design of Galileo scheme, in order to meet the stringent requirements of system availability, continuity and integrity, each orbital plane are emplaced a spare satellite. The spare satellite usually does not transmit signal, but drifts to complete the supplement under the control of the ground system when an operating satellites fails, ensuring the design specifications requirements [[2\]](#page-11-0).

In view of the spare satellites strategy's important effect, it is necessary to carry out the research on spare satellites strategy to ensure the system performance. It will provide a technical support for the long-term stable operation of the satellite navigation system. A method with spare satellites of navigation constellation is presented in this paper. First, learning from the GPS experience, and considering both the random failure and the wear-out failure, the improved satellite reliability is modelled. Second, the satellite reliability changes are analyzed and compared with the exponential model throughout the whole life. Then, the system availability is modelled and analyzed by using the Bayesian network. And then, combining the satellite reliability and the contribution of different satellites, the key satellite of the constellation can be identified with the enumeration method. Finally, considering the requirement of system availability, the rational spare satellites strategy which includes the spare mode, the spare satellite number, and the spare satellite launch plan is developed. Moreover, the simulation results can provide an important technical support for spare design alternatives trade-off and engineering decisions at the system operation stage.

28.2 System Availability Modeling

28.2.1 The Satellite Reliability Model

The satellite reliability which is the input condition of the spare satellites strategy is the key factor affecting the system stable operation and reflects it's the failure probability of the satellite. The satellite reliability's small changes may lead to the expected replacement satellite changing greatly. Even though the satellite reliability is known as statistical values, uncertainty changes also affect the anticipated replacement satellite time. The satellite replacement program risk level can be

evaluated by simulating the actual life of satellite random changes, thus this requires the precise satellite reliability model.

The satellite traditional reliability exponential model can simplify the analysis, but its hypothesis which the fault rate is constant contraries to the process of satellite waste accumulation and aging. In addition, the exponential distribution can not well describe the early failure and the failure depletion. In short, the reliability exponential model can not be a good reflection of the variation of the satellite reliability.

According to the GPS experience [\[3](#page-11-0)], an improved satellite reliability model is presented in this paper. The model is the product of the two models random fault model and depletion fault model. The random fault model mainly considers the random failures caused by the component failure, while the depletion fault model mainly considers the impact of loss device components.

The reliability function of the random fault model is expressed as:

$$
R_r(t) = e^{-(t/\alpha)^{\beta}} \tag{28.1}
$$

where α is the scale parameter related to the failure rate, β is the shape parameter determining the distribution, according to the different value of β , different failure rate curve can be obtained.

The reliability function of the depletion fault model is expressed as:

$$
R_w(t) = \int_{t}^{\infty} \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(t-\mu)^2}{2\sigma^2}} dt
$$
 (28.2)

where μ is the mean, σ is the standard deviation, and t is the usage time of the satellite.

So, the satellite reliability function is got:

$$
R(t) = R_r(t)R_w(t) \tag{28.3}
$$

While the satellite traditional reliability exponential model is:

$$
R(t) = e^{-\lambda t} \tag{28.4}
$$

where λ is the failure rate which is constant in the satellite's lifetime.

Figure 28.1 shows the comparison of the satellite improved reliability model and the satellite exponential model. It can be seen that the satellite improved reliability model takes into account the process of satellite waste accumulation and aging. The reliability is relatively stable during the satellite lifetime but with a sharp decline beyond the design life, which is more in line with the actual state of the satellite and better describes the satellite reliability.

Fig. 28.1 Curve: comparison of two reliability model of satellite

28.2.2 The Constellation State Probability

There are $(N + 1)$ kinds of possible states in the constellation of N satellites. S₀ is the state with no failure satellite, S_1 is the state with one failure satellite, and so on. Then the occurrence possibility of S_k is a probability value P_k . P_k is defined as the constellation state probability of k failure satellite. It determines the reliability of each satellite and all combinations, and there must be $\sum P_k = 1$. In order to describe the constellation state probability's dynamic changes over satellite reliability and adding the time information, it can be expressed as $P_k(t)$.

As the reliability of each satellite is different, the constellation state probability with different failure satellite combination is also different. There is

$$
\begin{cases}\nP_k(t) = \sum_{n=1}^{C_N^k} P_{k,n}(t) \\
P_{k,n}(t) = \left(\prod_{m=1}^{N-k} R_{n,m}(t)\right) \left(\prod_{i=N-k+1}^N (1 - R_{n,i}(t))\right)\n\end{cases} (28.5)
$$

where C_N^k is the combination number of k satellites in N, $P_{k,n}(t)$ is the constellation state probability of n case with k failure satellites, $R_{n,m}(t)$ is the reliability of each satellite of the $(N - k)$ satellites, and $(1 - R_{n,i}(t))$ is the corresponding failure probability.

28.2.3 The Constellation Performance Simulation Model

The constellations performance which reflects the pros and cons of the constellation configuration changes over time and space. In the process of the constellation design, the concerned indicators must be obtained by statistical methods, and optimized as the design goals or boundary conditions [\[4](#page-11-0)]. In the field of satellite navigation, the space signal accuracy is usually assumed to be known, so

it is generally designed basing on dilution of precision (referred to as DOP) value to measure the constellation performance.

According to GPS experience, the constellation value (referred to as CV) is chosen as the objective function to evaluate the constellation performance. CV which reflects the constellation geometry characteristics and continuous visibility is an important manifestation of the constellation performance.

CV is calculated as:

$$
CV = \frac{\sum_{t=t_0}^{t_0+\Delta T} \sum_{i=1}^{L} bool(DOP_{t,i} \le DOP_{\text{max}}) \times area_i}{\Delta T \times Area} \times 100\,\% \tag{28.6}
$$

where ΔT is the total simulation time, t_0 is the initial time, L is the total number of grid points, $bool(x)$ is Boolean function, when x is true, $bool(x) = 1$, when x is false, $bool(x) = 0, DOP_{t,i}$ is the DOP value of the *i* grid point at time *t*, Area is total service area, $area_i$ is the area of the *i* grid point.

28.2.4 The System Availability Model

The system availability is affected by two factors: constellation state and constellation configuration. In the constellation state aspect, due to the satellite reliability changes over time, the constellation state probability is also in dynamic change. In the constellation performance aspect, the constellation performance will be affected by the constellation configuration with failure satellite. Therefore, the system availability will vary with the changes of system operation time.

For generality, assuming the constellation as non-uniform configuration, then each state will have a different impact on the system performance. And the system can be calculated by the constellation state probability and the constellation value. There is:

$$
A(t) = \sum_{k=0}^{N} \sum_{n=1}^{C_N^k} P_{k,n}(t) \cdot \alpha_{k,n}(t)
$$
 (28.7)

where N is the total number of satellites, C_N^k is combination number of k failure satellites of N satellites, $P_{k,n}(t)$ is the constellation state probability with k failure satellites in *n* combination case, $\alpha_{k,n}(t)$ is the constellation value.

The Bayesian network model which is constituted by the network topology and the mathematical model (conditional probability tables, referred to as CPT) have the ability to express the complex relationship between the multi-state nodes (including the uncertainty relation). It is convenient to use Bayesian network for the system modelling, and to describe the influence relationship among satellites. And CPT is usually used to characterize it.

According to the constellation state probability model, it is narrowed to each satellite. That is to consider each satellite as a 'constellation'. Then the constellation state probability is transformed to the satellite reliability. CPT represents the constellation values of different failure satellites, which can be got by the constellation performance simulation.

Figure 28.2 shows the system availability model by using the Bayesian network.

It can be seen from Fig. 28.2 that in the system availability model, each satellite is parent node, and the reliability of each satellite is the marginal probability. Every node points to the ''system availability'' child node, and CPT is the constellation value of different failure satellites.

Assuming that the satellite reliability is $P(X_k)$, $k = 1, 2, ..., N$, which is also the marginal probability of the k node. The constellation value of different failure satellites can be got by the constellation performance simulation which is the conditional probability $P(X_{N+1}|parent(X_{N+1}))$. And parent (X_{N+1}) is the parent node set of "system availability" X_{N+1} . Using the Bayesian network chain rule, the joint probability distribution of all nodes is calculated as:

$$
P(X) = \prod_{k=1}^{N+1} P(X_k | parent(X_k))
$$
\n(28.8)

The Bayesian network inference mainly adopts the expression of the joint probability distribution. Given the observation (or evidence) of a set of variables E, that is the process of calculating the posterior probability distribution $(P(Q|E))$ of the set of variables Q . Researchers have made a lot of efficient algorithms for reasoning, including the exact algorithms and approximation algorithms [[5,](#page-11-0) [6\]](#page-11-0). Therefore, it is possible to calculate the 'system availability' under the conditions of knowing the reliability of each satellite.

Fig. 28.2 System availability model based on Bayesian network

28.3 Satellites Spares Strategy

The satellite reliability is a time-changing function, whose changes at different time may cause the changes of system availability. Based on the satellite reliability model, the system availability can be got by using the Bayesian network. Combined with the user demand, it can be judged to determine what time and which satellite to be supplemented. According to the value of system availability of user demand, it should be considered to launch spare satellites when the system doesn't meet the index requirements.

For uneven constellation, assuming that it consists of three types of orbits (the GEO, IGSO and MEO satellites). Considering the different launching time of each satellite, the contribution of the three types of satellites to the system availability is also different. So it should not simply replace the first launching satellite, but give a comprehensive consideration of the satellite launching time and the contribution to the system availability. In order to examine the contribution of a satellite to system availability at different time, the brute-force method is used to select the satellite needed spares to enhance the maximum system availability.

28.4 Simulation Examples

Without loss of generality, the non-uniform constellation is chosen for simulation. Assuming that there are 12 satellites of different orbits (e.g. S1–S12). Our country is defined as the service area. Position dilution of precision (referred to as PDOP) is considered as the criterion of the constellation simulation model, here PDOP threshold is supposed at 6. The satellite design life is set as 10 years, and the endof-life reliability is 0.6. The successful completion of constellation networking is within 2 years, and a satellite is launched in accordance with the serial satellite number every 6 months. Due to the satellite design life of 10 years, the launching satellite time of about 2 years, giving a comprehensive consideration of satellite life and launching satellite time, the spares satellite strategy will provide measures to maintain the system working in stable operation for 8 years. Then the constellation will be in the replacement stage, and the satellite reliability will be gradually restored and improved, so it no longer needs spares.

For fully and accurately reflecting the changes of each satellite reliability and system availability, the time of last launching satellite (that is the system networking success) is considered as the beginning time of computational analysis. According to the improved satellite reliability model, the reliability of every satellite at the time of system networking success can be got. Then the satellite reliability results will be listed after every 6 months, as shown in Table [28.1](#page-7-0). The changes of satellite reliability over time are as shown in Fig. [28.3.](#page-7-0)

It can be seen from Fig. [28.3](#page-7-0) that the earlier satellite is launched the lower reliability it is at the beginning time of system operation. The reliability is

Running time (year)		Satellite reliability										
	S ₁	S ₂	S ₃	S4	S5	S6	S7	S8	S ₉	S ₁₀	S ₁₁	S ₁₂
Ω	0.976	0.979	0.982		0.985 0.987	0.990			0.992 0.994 0.996 0.998		0.999	1.000
0.5	0.965	0.969	0.972		0.976 0.979		0.982 0.985 0.987		0.990	0.992	0.994	0.996
1.0	0.954	0.958	0.962		0.965 0.969		0.972 0.976		0.979 0.982 0.985 0.987			0.990
1.5	0.941	0.945	0.950		0.954 0.958 0.962 0.965 0.969 0.972 0.976 0.979							0.982
2.0	0.928		0.932 0.937		0.941 0.945 0.950 0.954 0.958 0.962 0.965 0.969							0.972
2.5	0.913	0.918	0.923		0.928 0.932 0.937 0.941						0.945 0.950 0.954 0.958	0.962
3.0	0.898	0.903	0.908	0.913	0.918	0.923	0.928		0.932 0.937	0.941	0.945	0.950
3.5	0.882	0.887	0.893	0.898	0.903	0.908	0.913	0.918	0.923	0.928	0.932	0.937
4.0	0.865	0.871	0.876		0.882 0.887	0.893		0.898 0.903		0.908 0.913	0.918	0.923
4.5	0.848	0.854	0.860		0.865 0.871		0.876 0.882 0.887			0.893 0.898	0.903	0.908
5.0	0.831	0.837	0.843		0.848 0.854 0.860 0.865 0.871					0.876 0.882 0.887		0.893
5.5	0.813	0.819		0.825 0.831 0.837 0.843 0.848 0.854 0.860 0.865 0.871								0.876
6.0	0.794	0.800	0.807		0.813 0.819						0.825 0.831 0.837 0.843 0.848 0.854	0.860
6.5	0.772	0.780	0.787	0.794	0.800	0.807	0.813		0.819 0.825 0.831		0.837	0.843
7.0	0.742	0.753	0.763		0.772 0.780	0.787	0.794	0.800	0.807	0.813	0.819	0.825
7.5	0.692	0.712	0.728		0.742 0.753	0.763	0.772	0.780	0.787	0.794	0.800	0.807
8.0	0.611	0.642	0.669		0.692 0.712	0.728			0.742 0.753 0.763	0.772 0.780		0.787

Table 28.1 Satellite reliability after system operation

Fig. 28.3 Curve: changes of satellite reliability after system operation

relatively stable during the satellite lifetime but with a sharp decline beyond the design life.

According to the constellation configuration and the constellation performance simulation results, the application of GeNIe software developed by the Decision Systems Laboratory of the University of Pittsburgh is used for the system availability modelling, as shown in Fig. [28.4.](#page-8-0)

Fig. 28.4 System availability model of a navigation constellation

Running time (year)	System availability	Running time (year)	System availability	Running time (year)	System availability
$\overline{0}$	0.9936	3.0	0.9518	6.0	0.8437
0.5	0.9894	3.5	0.94	6.5	0.8134
1.0	0.9842	4.0	0.9261	7.0	0.7745
1.5	0.978	4.5	0.91	7.5	0.7169
2.0	0.9706	5.0	0.8911	8.0	0.6201
2.5	0.9619	5.5	0.8692		

Table 28.2 Analysis results of system availability

Combined with the reliability of every satellite, the system availability analysis results can be calculated, as shown in Table 28.2.

And it is also described in curve, as shown in Fig. [28.5](#page-9-0).

Assuming the designed system availability is set at 0.9, and then it can be seen from Fig. [28.5](#page-9-0) that the system availability will drop to 0.8911 after 5 years which will no longer meet the target requirements. Therefore, in order to ensure the system's normal operation, to meet the user requirements, spare satellites should be launched to improve the satellite reliability, ensuring the system availability remaining above 0.9.

In order to examine the contribution of a satellite to system availability at different time, the brute-force method is used to select the satellite needed spares to enhance the maximum system availability. Results are shown in Table [28.3](#page-9-0) and described in Fig. [28.6](#page-9-0).

As can be seen from Table [28.3](#page-9-0) and Fig. [28.6](#page-9-0), after 5 years of system operation, it will get the maximum system availability to replace S1. After 6 years of system operation, it will get the maximum system availability to replace S4. After six point 5 years of system operation, it will get the maximum system availability to replace S3. After seven point 5 years of system operation, it will get the maximum system availability to replace S6. Therefore, it can be got the replaced satellite sequence: S1, S4, S3 and S6.

Fig. 28.5 Curve: changes of system availability after system operation

Table 28.3 Analysis results of system availability after substituting different satellites

Running time (year) System availability										
	S1	S2	S ₃	S4	S5	S6	S7	S8	$S9 - S12$	
5.0	0.918	0.909	0.911 0.91		0.908 0.91			0.906 0.908	0.896	
6.0	0.883	0.901	0.908 0.91			0.905 0.908 0.904 0.906 0.891				
6.5	0.893			0.915 0.923 0.893 0.912 0.922 0.917 0.919					0.902	
7.5	0.887	0.914		0.885 0.886 0.908 0.924 0.918				0.921	0.898	

Fig. 28.6 Chart: analysis results of system availability after substituting different satellites

According to the spares satellite program, the system availability analysis results are as shown in Table [28.4](#page-10-0) and described in Fig. [28.7](#page-10-0).

As can be seen from Table [28.4](#page-10-0) and Fig. [28.7,](#page-10-0) according to the spare program, the system availability is maintained at 0.8985 after 5 years of system operation, which could meet the requirements.

The above example shows that under the given reliability of every satellite, in order to meet the user requirements, it should be considered to replace the first launching satellite with S1 after 5 years of system operation. Then the second launching satellite should be replaced with S4 after 6 years of system operation.

Running time (year)	System availability				
$\mathbf{0}$	0.9936				
0.5	0.9894				
1.0	0.9842				
1.5	0.978				
2.0	0.9706				
2.5	0.9619				
3.0	0.9518				
3.5	0.94				
4.0	0.9261				
4.5	0.91				
5.0	0.918 (substituting S1)				
5.5	0.9016				
6.0	0.9102 (substituting S4)				
6.5	0.9226 (substituting S3)				
7.0	0.907				
7.5	0.9243 (substituting S6)				
8.0	0.8985				

Table 28.4 Analysis results of system availability based on spare program

Note 0.918 represents system availability after substituting S1

Fig. 28.7 Curve: results of system availability of spare program

After six point 5 years of system operation, the third launching satellite should be replaced with S3. Finally, the fourth launching satellite should be replaced with S6 after seven point 5 years of system operation. The spare program can ensure the system availability to meet the requirements in 8 years. According to the satellite design life, most of the satellites will go to the end of life after 8 years of system operation, and then it will need the constellation replacement.

28.5 Conclusions

The spare satellites strategy directly affects the realization of system availability, continuity and integrity, and is also the key factor to ensure the system working in stable operation when some satellite fails. In view of the important role of the spare satellites strategy, a rational spare program is proposed to ensure the system reliability to meet users' requirements in this paper.

First, learning from the GPS experience, and considering both the random failure and the wear-out failure, the improved satellite reliability is modelled. Second, the satellite reliability changes are analyzed and compared with the exponential model throughout the whole life. Then, the system availability is modelled and analyzed by using the Bayesian network. And then, combining the satellite reliability and the contribution of different satellites, the key satellite of the constellation can be identified with the enumeration method. And the spare satellite launch plan is developed. Finally, an example of a navigation constellation is given, which effectively verifies the system availability-based spare satellites method. And the simulation results can provide an important technical support for spare design alternatives trade-off and engineering decisions.

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